MATH-445 Measure and Integration

Credit Hours: 3-0

Prerequisite: MATH- 342 Real Analysis-II

Course Objectives: The course aims at an understanding of Lebesgue measure and integration and gives an alternative when Riemann integration fails. In this course the most fundamental concepts are presented: Sigma Algebra, Measurable Function, Outer Measure, Borel Measure, Lebesgue measure, Lebesgue integration and Lebesgue Differentiation.

Core Contents: Insufficiency of Riemann integrals, Outer Measure, σ -Algebra, Measurable sets and Measurable Space, Borel Sets & Borel Measure, S-Measurable function, Measure Space & Measure Functions, Lebesgue measure & Lebesgue Measurable functions, Egorov's Theorem, Lebesgue Integrals, Bounded Convergence Theorem, Dominated Convergence Theorem, L^1-Norm and Lebesgue spaces, Lebesgue Differentiation theorems, Lebesgue Density Theorem

Course Contents: Riemann intgerals, Insufficiency of Riemann integrals, Outer Measures, Properties of Outer Measure, σ -Algebra and its properties, Measurable Sets & Measurable Space, Borel Measure, Measurable Functions, Borel Measurable Functions, Properties of Borel Measurable Functions, S-measurable functions, Limit of S-measurable functions, Measures, Measure spaces, Measure preserving orders, Lebesgue Measure, Relationship among Measure, Borel Measure and Outer Measure and their properties, Lebesgue Measurable sets and properties, Egorov's Theorem and its conclusions, Lebesgue Measurable functions and its properties, Simple Functions, Lower and Upper Lebesgue Sum, Lebesgue integrations of Simple and Characteristic functions, Monotone Convergence Theorem, Lebesgue integration of some Real-valued functions and their properties, Bounded Convergence Theorem, Dominated Convergence Theorem, Relationship between Riemann integral and Lebesgue integrals, L^1-Norm and Lebesgue spaces, Lebesgue Differentiation theorems, Lebesgue Density Theorem

Course Outcomes: Upon completion of this course, the student should be able to:

Recognize Insufficiency of Riemann integrals, Outer Measure, σ -Algebra

Borel Measure, Borel Measurable Functions, S-measurable functions, Lebesgue Measure

Egorov's Theorem, Lower and Upper Lebesgue Sum, Lebesgue integrations of Simple and Characteristic functions

Bounded Convergence Theorem, Dominated Convergence Theorem, Lebesgue Differentiation theorems, Lebesgue Density Theorem

Text Book: S. Axler, "Measure, Integration & Real Analysis", Springer, (2020).

Reference Books:

S. J. Taylor, “Introduction to Measure and Integration”, Cambridge University Press, (2010).

M. T. Nair, “Measure and Inegration: A first Course”, Taylor & Francis Group, NY, (2019)

C. S. Kubrusly, “Essentials of Measure Theory”, Springer, (2015)

Weekly Breakdown		
Week	***Section***	***Topics***
1	Sec. 1A-1B	Review of Riemann Integral, Insufficiency of Riemann Integral and Examples
2	Sec. 2A	Outer Measure on R, Examples, Properties of Outer Measure, Outer Measure on Closed Intervals
3	Sec.2A-2B	Non-Additive Outer Measure and Examples, σ -Algebra, examples and properties, Measurable sets, and Measurable Spaces
4	Sec. 2B	Borel Sets, Examples, Direct & Inverse Image of σ -Algebra and their properties, Measurable Functions and Borel Measurable Functions
5	Sec. 2B-2C	Relationship between Continuity and Borel Measurable functions, Algebraic Properties of Borel Measurable Functions, S-measurable functions, Limit of S-measurable functions, Measures and Examples
6	Sec. 2C	Measure spaces, Measure preserving orders, measure of an increasing union, measure of a decreasing intersection, Countable Subadditivity properties of Measure spaces
7	Sec. 2D	Lebesgue Measure, Relationship among Measure, Borel Measure and Outer Measure and their properties
8	Sec. 2D-2E	Lebesgue Measurable set, examples and properties, Comparison of Pointwise and Uniform convergence
9	**Mid Semester Exam**	
10	Sec. 2E	Egorov’s Theorem and its conclusions, Lebesgue Measurable functions and its properties. Simple Functions, Relationship between Lebesgue measurable functions and Borel Measurable functions
11	Sec. 3A	Lower and Upper Lebesgue Sum, Examples, Lebesgue integrations of Simple and Characteristic functions, Monotone Convergence Theorem
12	Sec. 3A-3B	Integral -type sums for simple functions, Lebesgue integration of some Real-valued functions and their properties, Bounding a Lebesgue integral,
13	Sec. 3B	Bounded Convergence Theorem, almost everywhere convergence, Dominated Convergence Theorem, Relationship between Riemann integral and Lebesgue integrals

14	Sec. 3B	L1-Norm and Lebesgue spaces, Examples and their properties.
15	Sec. 4A-4B	Markov's inequality, Hardy-Little maximal inequality, Lebesgue Differentiation theorem First Version
16	4B	Lebesgue Differentiation Theorem second version, Density in Lebesgue spaces, Lebesgue Density Theorem
17		Review
18	**End Semester Exam**	